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Polaron in a superlattice

P V Panat† and V V Paranjape‡

† Department of Physics, University of Poona, Pune, 411007 Maharastra, India

‡ Department of Physics, Lakehead University, Thunder Bay, Ontario P7B 5E1, Canada

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Abstract. The object of the present paper is to calculate the mass and the ground-state energy of a polaron in a superlattice which is formed by alternate layers of materials such as GaAs and $Ga_{1-x}Al_xAs$. We obtain the variations in these two quantities as a function of the separation between the layers and the electron concentration. The calculations are performed using the zero-point energy approach.

1. Introduction

The calculations of the mass and the ground-state energy of the polaron in a two-dimensional (2D) and quasi-2D electron gas have been reported by several workers [1–4]. In [1–3], the effect of electron screening on the electron–phonon interaction is examined. It is shown that the electron screening has a significant effect on the electron–phonon interaction in 2D. In a superlattice the screening effects are more complex than the corresponding effects in a pure 2D or 3D electron gas. In a superlattice the screening of the electron–lattice interaction can occur because of electrons in a single layer if the separation between the layers is larger than the polaron radius, and because of electrons in several layers if the separation is smaller. In addition, the effect of the electron screening on the electron–phonon interaction can be altered by selecting the electron concentration in each layer by adjusting the doping. Thus a superlattice provides an interesting system to study the effect of the electron gas on the polaronic properties.

For our calculations we assume that the superlattice is formed by layers of a 2D electron gas separated by a distance a. This is an idealization of a real situation in which the electron wavefunction has a spread in the direction normal to the 2D plane. Since our main object is to study the effect of the superlattice, the assumption that each layer is a pure 2D electron gas is unlikely to affect the features arising from the properties of the superlattice. With this assumption the electrons are restricted to their own layer. They are, however, allowed to have a Coulomb interaction with electrons in their own layer as well as with electrons in other layers. We have also made the assumption that the electron–lattice interaction in a superlattice is the same as that found in an equivalent 3D crystal. The assumption, also made by other workers [1–3], is reasonable since the ionic motion is almost the same in both the materials which form the superlattice.

The method of calculations adopted in this paper is somewhat unconventional in the sense that it is not based on the use of the Fröhlich Hamiltonian. We use the zero-point energy formalism which was developed earlier by Hawton and Paranjape [5] for the

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study of the polaron in 3D and by Panat and Paranjape [3] for the 2D polaron. In this formalism we calculate the self-energies of the electron in the presence and in the absence of the electron–phonon interaction. The difference between the two energies provides the value of the electron–phonon interaction. The effect of the screening is introduced through the dielectric constant which contains contributions from the lattice and the layered electron gas. Our approach, we believe, is simpler and more direct than the approaches based on the Fröhlich Hamiltonian. Apart from some numerical differences the two methods would provide similar results for the polaron mass.

The details of our model are given in section 2, the results in section 3 and a discussion of the results in section 4.

2. Model

In the zero-point energy formalism, we calculate the difference between the self-energy of an electron in the presence and that in the absence of the electron–phonon interaction. The difference provides the interaction energy.

Consider macroscopic electric fields $E(r, \omega)$ at point r and $E(r', \omega)$ at point r'. The two fields are connected to each other by the relation

$$E(r, \omega) = \int \mathbf{F}(r, r', \omega) E(r', \omega)\, \mathrm{d}^3 r' \tag{2.1}$$

where the tensor $\mathbf{F}$ depends on the properties of the medium. In the absence of the medium, $\mathbf{F}$ is directly given by $\delta(r - r')$. Upon taking the Fourier transform, equation (2.1) becomes

$$E(k, \omega) = \sum_{k'} \mathbf{F}(k, k', \omega) E(k', \omega). \tag{2.2}$$

Equation (2.2) provides the eigenmodes of the system. The zero-point energy of these modes is given by Mahanty and Paranjape [6] according to

$$E = \frac{\hbar}{4\pi \mathrm{i}} \oint \mathrm{d}\omega \, \mathrm{Tr} \left(\sum_g \frac{[\mathbf{F}(k, k', \omega)]^g}{g} \right) \tag{2.3}$$

where g is the summation index which takes positive integral values starting from unity and the contour encloses the real axis of the complex ω-plane. For linear effects it is sufficient to consider the $g = 1$ term only. If we neglect the electron–phonon interaction, then the zero-point energy of the system is given by equation (2.3) provided that we replace $\mathbf{F}$ by $\mathbf{F}_0$ which includes properties of the medium from which the effect of the electron–phonon interaction has been excluded. The difference ΔE between the two energies gives the self-energy of the electron due to its interaction with the lattice:

$$\Delta E = \frac{\hbar}{4\pi \mathrm{i}} \oint \mathrm{d}\omega \, \mathrm{Tr}[\mathbf{G}(k, k', \omega)] \tag{2.4}$$

where $\mathbf{G}$ is the difference between $\mathbf{F}$ and $\mathbf{F}_0$. The expression for $\mathbf{G}$ has been derived by Hawton and Paranjape [5] for an electron in a 3D crystal. The derivation is long but straightforward. The main idea in the derivation is as follows. We consider a macroscopic electric field $E(r, \omega)$ at point r and find the effect of the field on the wavefunction of a designated electron using the first-order time-dependent perturbation theory. The

perturbed wavefunction is used to calculate the average value of the electric field $E(r', \omega)$ at point r' using the standard quantum mechanical averaging procedure. The result obtained is then compared with equation (2.1). The comparison allows us to determine the expressions for $\mathbf{F}(r, r', \omega)$ and $\mathbf{F}(k, k', \omega)$. The effect of the medium is brought into the picture through the use of the dielectric constant in obtaining the average value of the electric field. With a suitable choice of the dielectric constant, it is possible to determine $\mathbf{F}$ and $\mathbf{F}_0$ and hence $\mathbf{G}$. The expression for $\mathbf{G}$, as given by Hawton and Paranjape [5] for a 3D crystal, can be easily modified for a superlattice. The resultant expression for an electron in a superlattice is given by

$$\mathbf{G}(k, k', \omega) = \frac{(2\pi)^3}{V}\frac{e^2}{\hbar(2\pi)^2}\sum_{k'', l''}\frac{k \cdot k'}{k^2 k'^2}\left(\frac{1}{\varepsilon_{\text{tot}}(k, \omega)} - \frac{1}{\varepsilon_{\text{tot}}(k, \infty)}\right)$$
$$\times\left(\frac{\langle k_0, l_0|\exp(-ik' \cdot r)|k'', l''\rangle\langle k'', l''|\exp(ik \cdot r)|k_0, l_0\rangle}{\omega_{k'', k_0} - \omega}\right.$$
$$\left. + \frac{\langle k_0, l_0|\exp(ik \cdot r)|k'', l''\rangle\langle k'', l''|\exp(-ik' \cdot r)|k_0, l_0\rangle}{\omega_{k'', k_0} + \omega}\right) \tag{2.5}$$

where V is the periodic volume. In equation (2.5) the electron wavefunction in the l_0th layer is given by

$$\langle r|k_0, l_0\rangle = (1/A)^{1/2}\exp(ik_0 \cdot r)\,\varphi(z - l_0 a) \tag{2.6}$$

where k_0 is the 2D wavevector of the electron in the plane of the l_0th layer, A is the periodic area and $\varphi(z - l_0 a)$ is the electron wavefunction in the direction normal to the 2D plane. A similar expression can be defined for the intermediate states of the electron. We choose the form of the wavefunction $\varphi(z - l_0 a)$ such that

$$\varphi^*(z - l_0 a)\varphi(z - l'' a) = \delta_{l_0 l''}\delta(z - l_0 a) \tag{2.7}$$

so that the electron is restricted to a single layer defined by the wavefunction and that there is no overlap between wavefunctions belonging to different layers. Further, in equation (2.5), $\omega_{k'', k_0} = \hbar[(k'')^2 - (k_0)^2]/2m$, where m is the band mass and we have neglected the excited states occurring in the direction normal to the 2D surface. It is important to note that in equation (2.5) the wavectors k_0 and k'' are associated with the electron states and are therefore two dimensional, but vectors k and k' in view of their origin are vectors in 3D. The difference between the dielectric constants occurring in equation (2.5) arise because of the differences in the definitions of $\mathbf{F}_0$ and $\mathbf{F}$.

According to Mahan [7], the total dielectric constant of the medium due to all sources can be written as follows:

$$\varepsilon_{\text{tot}}(k, \omega) = \varepsilon_\infty + (\varepsilon_0 - \varepsilon_\infty)/[1 + (\omega/\omega_t)^2] - v(k)\pi(k) \tag{2.8}$$

where the first term is the contribution to the dielectric constant due to the electrons of the atomic core of the crystal, the second term is the contribution of the vibrating lattice with ω_t as the transverse vibrational frequency, and the third term is the contribution of the layered electron gas. Here $v(k)$ is the 2D Fourier transform of the Coulomb potential and $\pi(k)$ is the polarizability of the electron gas. We have considered the static response of the electron gas. The justification for its use has been given by Das Sarma and Mason [2]. In our opinion the static response of the electron gas provides at least a reasonable

estimate of the screening, and its use is considered as a simplifying assumption. Solving for the inverse of the dielectric constant we get

$$1/\varepsilon_{\text{tot}}(k, \omega) = (\omega_t^2 - \omega^2)/\varepsilon_\infty \mu_\infty(k)(\omega_l^2 - \omega^2) \tag{2.9}$$

where

$$\omega_l^2 = \omega_0^2 \mu_0(k)/\mu_\infty(k) \tag{2.10}$$

with

$$\mu_0 = 1 - v(k)\pi(k)/\varepsilon_0 \qquad \mu_\infty = 1 - v(k)\pi(k)/\varepsilon_\infty. \tag{2.11}$$

The expressions for $v(k)\pi(k)$ for the layered electron gas in a superlattice have been given by Das Sarma and Quinn [8]. Using their expressions allows us to write

$$\mu_i = \begin{cases} 1 + (2me^2/\hbar^2 k\varepsilon_i)\{\sinh(k_\parallel a)/[\cosh(k_\parallel a) + \cos(k_\perp a)]\} \\ 1 + (2me^2/\hbar^2 k\varepsilon_i)\{1 - [1 - (8\pi n/k_\parallel^2)]^{\frac{1}{2}}\} \sinh(k_\parallel a) \\ [\cosh(k_\parallel a) + \cos(k_\perp a)] \end{cases} \quad \text{for} \begin{cases} k_\parallel^2 < 8\pi n \\ k_\parallel^2 > 8\pi n \end{cases} \tag{2.12}$$

where a is the distance separating the layers of the superlattice, $k_\parallel$ is the component of k parallel to the 2D plane, $k_\perp$ is the perpendicular component and n is the electron concentration per unit area of the layer. In equation (2.10), ω_0 is the longitudinal optical frequency of the lattice and is related to ω_t by the relation $\omega_0^2/\omega_t^2 = \varepsilon_0/\varepsilon_\infty$.

3. Results

We now obtain the matrix elements occurring in equation (2.5) by making use of the electron wavefunction defined by equations (2.6) and (2.7). On performing the summation over the intermediate states by utilizing the properties of the delta function, we can rewrite equation (2.5) as

$$\mathbf{G}(k, k', \omega) = [(2\pi)^3/V][e^2/\hbar(2\pi)^2][(k \cdot k')/k^2 k'^2]\{[1/\varepsilon_{\text{tot}}(k, \omega) - 1/\varepsilon_{\text{tot}}(k, \infty)]\} \times [1/(\omega_{|k_0 + k'|, k_0} - \omega) + 1/(\omega_{|k_0 - k|, k_0} + \omega)]\delta_{k_\parallel, k'_\parallel} \exp[-i(k_\perp - k'_\perp)l_0 a] \tag{3.1}$$

where the intermediate states, in view of the degeneracy of the electron gas, are above the Fermi level. Hence $|k_0 + k'| > k_F$ and $|k_0 - k| > k_F$ where k_F is the Fermi wavevector of the 2D electron gas. If we now take the trace of $\mathbf{G}$, we get

$$\text{Tr}[\mathbf{G}(k, k, \omega)] = \frac{2\pi}{V}\frac{e^2}{\hbar}\sum_k \frac{1}{k_\parallel^2 + k_\perp^2}\left(\frac{1}{\varepsilon_{\text{tot}}(k_\parallel, k_\perp, \omega)} - \frac{1}{\varepsilon_{\text{tot}}(k_\parallel, k_\perp, \omega)}\right) \times \frac{2\Omega_{k_\parallel}}{\Omega_{k_\parallel}^2 - \omega^2} \tag{3.2}$$

where

$$\Omega_{k_\parallel} = (\hbar/2m)(k_\parallel^2 + 2k_\parallel k_0 \cos\Phi) \tag{3.3}$$

and Φ is the angle between k_0 and $k_\parallel$. In writing equation (3.2), we have replaced k by $-k$ in the term $1/(\omega_{|k_0 - k|, k_0} + \omega)$ in equation (3.1). The replacement is possible since the summation in equation (3.1) is over all values of k. We now substitute equation

(3.2) into equation (2.4), using equations (2.9) and (2.10). On performing the contour integration and changing the summation over k to integration we get

$$\Delta E = -\frac{e^2\omega_o}{4\pi^2\varepsilon_\infty}\int\int \frac{d^2k_\parallel\, dk_\perp}{(k_\parallel^2+k_\perp^2)}\frac{\varepsilon_\infty/\varepsilon_0-\mu/\mu_\infty}{(\mu_\infty\mu_0)^{1/2}[(\hbar/2m)(k_\parallel^2+2k_0k_\parallel\cos\Phi)+\omega_o(\mu_0/\mu_\infty)^{1/2}]}. \tag{3.4}$$

The variation in $k_\perp$ is from $-\infty$ to $+\infty$, while the variation in $k_\parallel$ is prescribed by $(2\pi n)^{1/2} < k_\parallel < \infty$. This prescription arises for two reasons: all the intermediate states in (2.5) are above the Fermi level, and k_0 is close to zero. For k_0 approaching zero (i.e. the self-energy of the electron is for those electrons which are near the ground state), we may expand equation (3.4) in powers of k_0 and consider terms in the expansion up to square in k_0^2. We also write the integral in (3.4) in a dimensionless form by substituting $x = k_\parallel a_p$ and $y = k_\perp a_p$, where a_p is the polaron radius given by the well known expression $a_p = (\hbar/2m\omega_o)^{1/2}$. If we now write ΔE in (3.4) as

$$\Delta E = \Delta E_0 + \Delta E_1 \tag{3.5}$$

then

$$\Delta E_0 = -\frac{\alpha\hbar\omega}{\pi}\int\int\frac{x\,dx\,dy}{x^2+y^2} \times \frac{\varepsilon_\infty/\varepsilon_0-\mu_0(x,y)/\mu_\infty(x,y)}{(1-\varepsilon_\infty/\varepsilon_0)[\mu_0(x,y)\mu_\infty(x,y)]^{1/2}\{x^2+[\mu_0(x,y)/\mu_\infty(x,y)]^{1/2}\}} \tag{3.6}$$

and

$$\Delta E_1 = -\frac{\alpha\hbar^2k_0^2}{m}\int\int\frac{x^3\,dx\,dy}{x^2+y^2}\frac{\varepsilon_\infty/\varepsilon_0-\mu_0(x,y)/\mu_\infty(x,y)}{1-\varepsilon_\infty/\varepsilon_0} \times \frac{1}{[\mu_0(x,y)\mu_\infty(x,y)]^{1/2}\{x^2+[\mu_0(x,y)/\mu_\infty(x,y)]^{1/2}\}^3} \tag{3.7}$$

where we have used the standard definition for the Fröhlich coupling constant α given by

$$\alpha = \tfrac{1}{2}(1/\varepsilon_\infty - 1/\varepsilon_0)(e^2/\hbar\omega_o)(2m\omega_o/\hbar)^{1/2}.$$

We now define the polaron mass m_{pol} according to

$$1/m_{pol} = 1/m + (1/\hbar^2)[\partial^2(\Delta E)/\partial k_0^2] \tag{3.8}$$

then

$$\frac{m_{pol}-m}{m} = \frac{\Delta m}{m} = \frac{2\alpha}{\pi}\int\int\frac{x^3\,dx\,dy}{x^2+y^2}\frac{\varepsilon_\infty/\varepsilon_0-\mu_0(x,y)/\mu_\infty(x,y)}{1-\varepsilon_\infty/\varepsilon_0} \times \frac{1}{[\mu_0(x,y)\mu_\infty(x,y)]^{1/2}\{x^2+[\mu_0(x,y)/\mu_\infty(x,y)]^{1/2}\}^3}. \tag{3.9}$$

Equations (3.6) and (3.9) represent the main results of this paper.

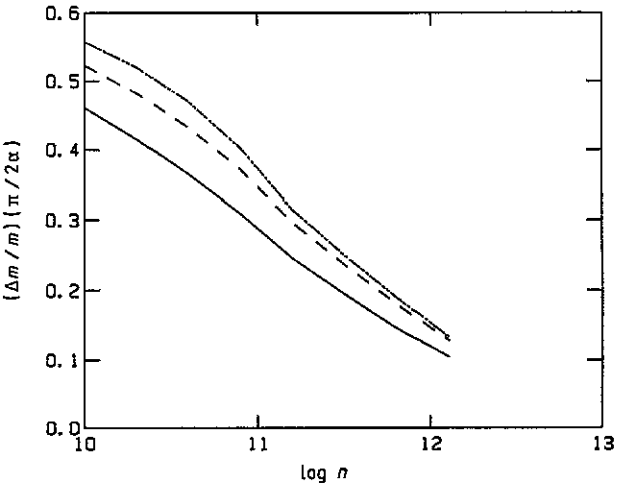

Figure 1. $(\Delta m/m)(\pi/2\alpha)$ as a function of n (the electron concentration per unit area of the layer) for various distances a of separation between the layers: ——, $a = 10$ Å; – – –, $a = 30$ Å; —·—·—, $a = 90$ Å.

4. Discussion

In this paper, we have considered an ideal superlattice consisting of sheets of electron gas separated by a fixed distance which is defined as the lattice constant of the superlattice. The electrons are allowed to move within a sheet, but electron transitions between the sheets are not allowed. The interaction between an electron and the longitudinal optical phonon modes is studied on the assumption that the interaction Hamiltonian is the same as in the corresponding 3D crystal. Making use of these assumptions, we have studied the effect of the electron screening on the electron–phonon interaction. The effect of the screening on the ground-state energy and the effective mass of the polaron is calculated and the results are given by equations (3.6) and (3.7). The integrals in these two equations has been evaluated numerically and the variations in $(\Delta m/m)(\pi/2\alpha)$ and $\Delta E_0/\alpha\hbar\omega_0$ with the electron concentration per unit area of a layer of the superlattice are shown in figures 1 and 2, respectively. In each of these figures we have also considered three values for the separations between the layers of the superlattice. In the numerical calculations we have assumed the following values which are appropriate for the $Ga_{1-x}Al_xAs$ system: $\varepsilon_\infty = 10.9$, $\varepsilon_0 = 12.82$, $m/m_0 = 0.0665$, where m_0 is the electron mass in free space, and $\alpha = 0.07$.

The effect of the electron gas on the polaronic properties occurs in two ways. One effect is due to the degeneracy of the electron gas. The electron is unable to make virtual transitions to the intermediate states with an energy less than the Fermi energy. This effect puts a lower limit in the integration over k in equation (3.4). The direct effect of the screening on the electron–phonon interaction is contained in the factors μ_0 and μ_∞ defined by equation (2.12). The combined effect of these two has been included in our numerical evaluations.

If we let the electron concentration in a layer approach zero, then both expressions for μ approach unity and the effect of the electron gas on the polaron energy vanishes. Putting the μ-values equal to unity in equations (3.6) and (3.9), we are able to evaluate

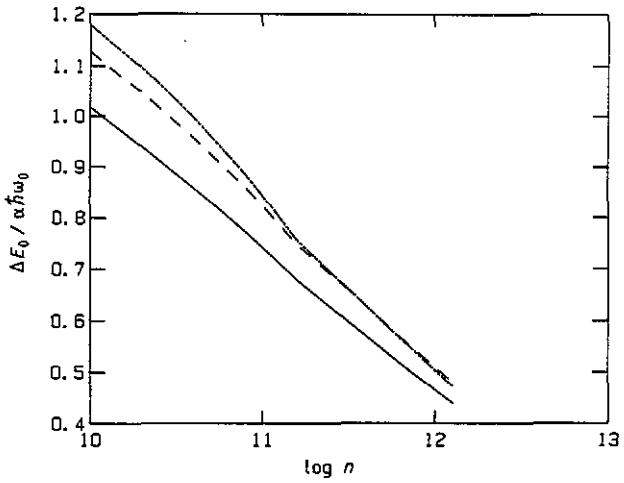

Figure 2. $\Delta E_0/\alpha\hbar\omega_0$ as a function of n (the electron concentration per unit area of the layer) for various distances a of separation between the layers: ——, $a = 10$ Å; – – –, $a = 30$ Å; —·—·—, $a = 90$ Å.

analytically the integrals in these expressions giving $(\Delta E_0/\alpha\hbar\omega_0) = \pi/2 \simeq 1.571$ and $(\Delta m/m)(\pi/2\alpha) = \pi^2/16 \simeq 0.617$. These values are in agreement with the work of Paranjape and Panat [4]. The numerical results for $(\Delta m/m)(\pi/2\alpha)$ for a separation $a =$ 90 Å and for $n = 10^{10}$ cm^{-2} is 0.56. This value differs from the theoretical value in which the effect of the electron gas is neglected by about 10%. We conclude that, for this choice of n and of a, the effect of the electron gas on the polaron mass is quite small. For the $Ga_{1-x}Al_xAs$ system the polaron radius is of the order of 38 Å; the effect of the superlattice decreases as the separation between the layers increases beyond the polaron radius. This is evident from both the figures in which the difference between the two curves for $a = 30$ and $a = 90$ is already small and is expected to decrease further as a is increased.

For the ground-state energy of the polaron the effect of the electron gas is still quite strong even at an electron concentration of 10^{10} cm^{-2} as is seen from figure 2. As the separation between the layers is increased, the effect of the superlattice diminishes and the polaron properties are similar to that of a 2D polaron. On the other hand, if the separation between the layers is decreased, the superlattice possesses properties which are intermediate between those of a 2D and a 3D crystal. A superlattice, in view of the restricted motion of the electrons, cannot approach the properties of a 3D crystal even when the distance between the layers is made very small.

We wish to comment on the limitations of the model which we have used in this paper. There are two main drawbacks. The first is the assumption that the superlattice is formed by sheets of 2D electron gas, and the second is that the electron–phonon interaction Hamiltonian is the same as in the corresponding 3D crystal. These objections are valid in most cases, since the superlattices produced in laboratories consist of electrons trapped in quasi-2D quantum wells which have small but finite widths. As a consequence the electron wavefunctions in adjacent layers can overlap, which is contrary to the assumption of our model. In addition, the frequencies and dispersions of the lattice vibrations (phonons) within the well, on the walls of the well, and in regions outside the well differ from each other and from their 3D behaviour. We have neglected

these differences. Although we recognize these limitations, we have tried to represent a relatively simple model in which the main emphasis is on calculating the effect of the screening of the electron–phonon interaction in a superlattice. To achieve simplicity, we have excluded factors which we think are extraneous to the main aim of the paper.

Recently, Guo-Qiang Hai *et al* [9] and Mari and Ando [10] have examined the effect of the localized phonons on the electron–phonon interaction in a quantum well. Neglecting the electron screening effects, Guo-Qiang Hai *et al* [9] have also made a comparative study of the effect of the localized phonon modes and the 3D phonon modes on the mass and the ground-state energy of the polaron for the $GaAs/Ga_{1-x}Al_xAs$ quantum well. They find that the differences in the mass and energy due to the considerations of the localized phonons and the 3D phonons decrease very rapidly as the width of the well is increased. Using the localized phonons, these workers show that in an infinite potential well of width greater than 20 Å the polaron mass differs from its 3D phonon value by less than 10%. For the case of a finite potential well the difference between the two results vanishes at zero width, increases to a peak value at approximately 10 Å and decreases rapidly as the width is further increased. At its peak the difference between the values of the polaron mass obtained using localized and 3D phonons is about 10% for $x = 1$. Similar results also hold for the ground-state energy of the polaron.

In our calculations we have assumed the width of the well to be zero and have used the 3D model for the phonons. The effects of these two assumptions on the mass and the ground-state energy occur in opposite directions. As the width of the well is increased, the polaron mass decreases; on the other hand, replacement of the 3D phonons by the localized phonons produces an increase in the polaron mass. Consequently in $GaAs/Ga_{1-x}Al_xAs$, for an infinite potential well of width of about 20 Å, the polaron mass obtained by using the localized-phonon model is the same as the polaron mass obtained using the 3D phonons in a zero-width well. The effect of the localized phonons in general is to increase the polaron mass from its value obtained using the 3D phonons. It is not possible to predict with great accuracy the effect of the replacement of 3D phonons by localized phonons on the results of this paper, but it is reasonable to expect that the mass and the energy of the polaron would be increased by about 10–20%. However, the form of the variation in the mass and the energy of the polaron due to the screening effect of the electron gas is unlikely to be affected significantly by the consideration of the localized phonons.

Furthermore the bound-state energy levels in a narrow quantum well are separated by a sufficiently large amount that it is usually adequate to assume that only the lowest bound energy level is occupied by the electrons, and it is also common practice to neglect the virtual transitions of the electrons to the higher levels as a result of the electron–phonon interaction. Guo-Qiang Hai *et al* [9] have studied the effect of virtual transitions to all higher levels and find that for small well widths the energy and mass of the polaron are not significantly affected by the assumption. For the purposes of this paper the assumption is not serious if we restrict ourselves to superlattices with small quantum well widths.

As the width of the barrier is decreased, additional changes occur which are neglected in this paper. The electron wavefunctions in adjoining wells would increasingly overlap as the width of the barrier is reduced. The electron wavefunctions which we assumed to be restricted to a layer must be replaced by the extended electron wavefunctions in the direction normal to the superlattice. The proposed replacement in equations (2.5), (2.6) and (2.7) would affect the conclusions of this paper. We propose to study these effects in the future. The results of this paper are therefore restricted to superlattices in which

the overlap is negligible or is sufficiently small. The energy and mass of the polaron as functions of electron concentration are given for superlattice separations of 10, 30 and 90 Å in figures 1 and 2. The theoretical curves for the three separation values are given to show the differences predicted by the theory for small, average and large separations. These curves represent the results of our theory. The effect of the electron overlap would affect the results shown in figures 1 and 2 for the smallest separation but the results for the other two separations may not be significantly affected.

Finally we wish to remark that the polaron mass is known to be measurable in ionic crystals to a great degree of accuracy. The major limiting factor in the measurement of the polaron mass is the purity of the sample. We hope that these measurements can be performed in a superlattice. $Ga_{1-x}Al_xAs$ is not ideal for studying the superlattice effect on the polaron mass since the Fröhlich coupling constant in this material is small and so also are the polaronic effects in this material. More suitable superlattices would be those made with materials having a larger value of α.

Acknowledgment

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